ANALOGIES : CHAPTER 4

- Equations of motion and the behavior of systems involving various physical media are analogous.

- Electrical analogy for thermal, fluid, and mechanical systems are developed.

- Network diagrams are used conveniently employing techniques already available from electrical-network theory

Analogies between physical media

* Strong analogies between the equations of motion for lumped physical models of systems in the different physical media.

Electrical Analog of Heat Conduction

Thermal capacitance $\approx$ Electrical capacitance : C
Thermal resistance $\approx$ Electrical resistance : R

Variables

Through variables : Heat flow q $\approx$ Electrical current: i
Across variables : Temperature T $\approx$ Voltage: v.
Constraints : Prescribed T (at a boundary) $\approx$ Prescribed voltage v (at a node)
Environment $\equiv$ Temperature source.
Prescribed heat flow q (through a layer)
$\approx$ prescribed i (through a branch)

Equilibrium

First law of thermodynamics without work

$$\frac{dT}{dt} = \frac{1}{C} q_{net}$$

(applied at a junction without heat storage) $\approx$ KCL

Temperature compatibility around a path $\approx$ KVL

<u>Physical relations</u>

Resistance : $q = \frac{1}{R}(T_a - T_b)$; $\quad i = \frac{1}{R}(v_b - v_a)$

Capacitance: $q = C\dot{T}$; $\quad i = C\dot{v}$

* No thermal analog for inductance !

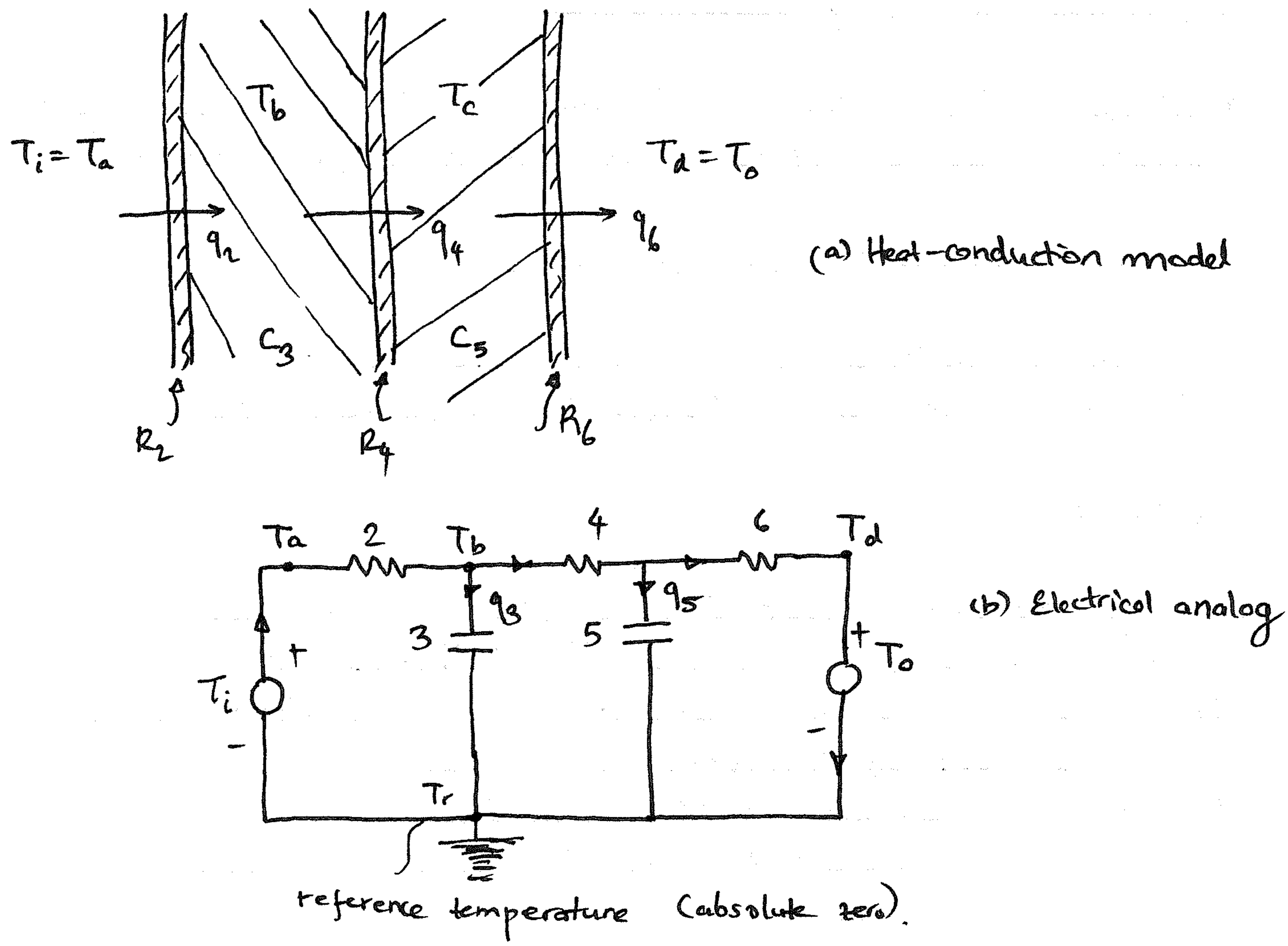


Figure. Electrical analog of a heat-conduction system.

* Expert in one field can learn the other more easily
* It is possible to construct a real electric circuit

whose dynamic behavior will be the same as that of a given heat-flow model.

Electrical Analogs of Fluid Systems

- Lumped fluid systems only can be represented analogously by electrical networks.

- Body inertia forces are neglected.

Equations of motion:

Variables and geometry:

Through variables : fluid flow rate w $\Rightarrow$ current i
Across variables : pressure p (or reservoir height h) $\Rightarrow$ voltage v

Equilibrium : Continuity of flow $\Rightarrow$ KCL

Physical relations:

Resistance:

$$w = c_\alpha (p_1 - p_2)^{1/\alpha} \quad \& \quad w = c'_\alpha (h_1 - h_2)^{1/\alpha}$$

$$\Rightarrow \quad i = \frac{1}{R} v \quad \text{"Ohm's law"} \qquad (\alpha = 1)$$

$\alpha \neq 1$ suggests $\quad i = c\, v^{1/\alpha}$.

Capacitance:

$$\dot{p} = \frac{1}{C} w_{net} \quad \& \quad \dot{h} = \frac{1}{C} w_{net}$$

$$\Rightarrow \quad \dot{v} = \frac{1}{C} i$$

Electrical Analog of Mechanical Systems

Two analogies are possible

u (velocity) $\longrightarrow$ v (voltage)
f (force) $\longrightarrow$ i (current)

} Consistent with the previous developments since through and across variables are mapped to the through and across variables!

or

$f \rightarrow v$ and $u \rightarrow i$

Variables

Through variables: $f \rightarrow i$
Across variables: $u \rightarrow v$

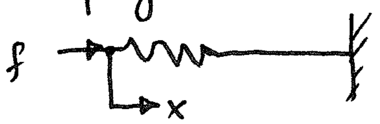
Equilibrium

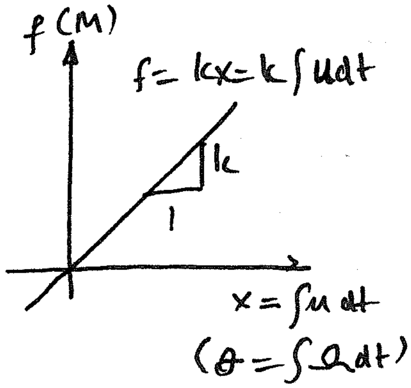
$$\Sigma f^* = 0 \qquad \Sigma M^* = 0 \quad \approx \quad \Sigma i = 0 \text{ (for a node)}$$

Geometric compatibility $\approx$ KVL.

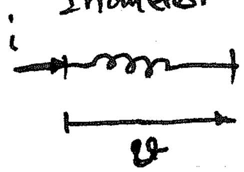
Physical relations

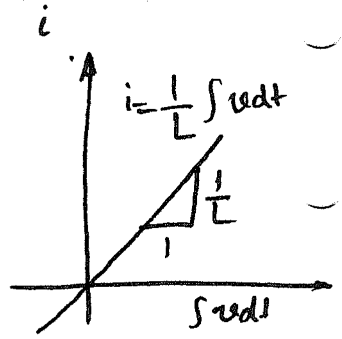
spring



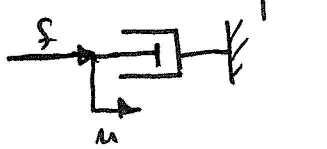


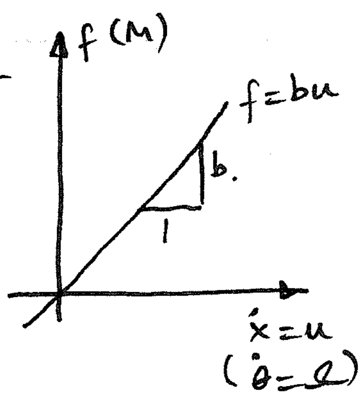
Inductor



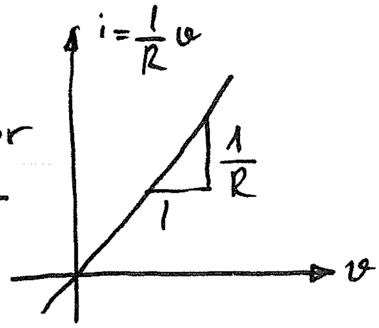


Viscous damper





Resistor



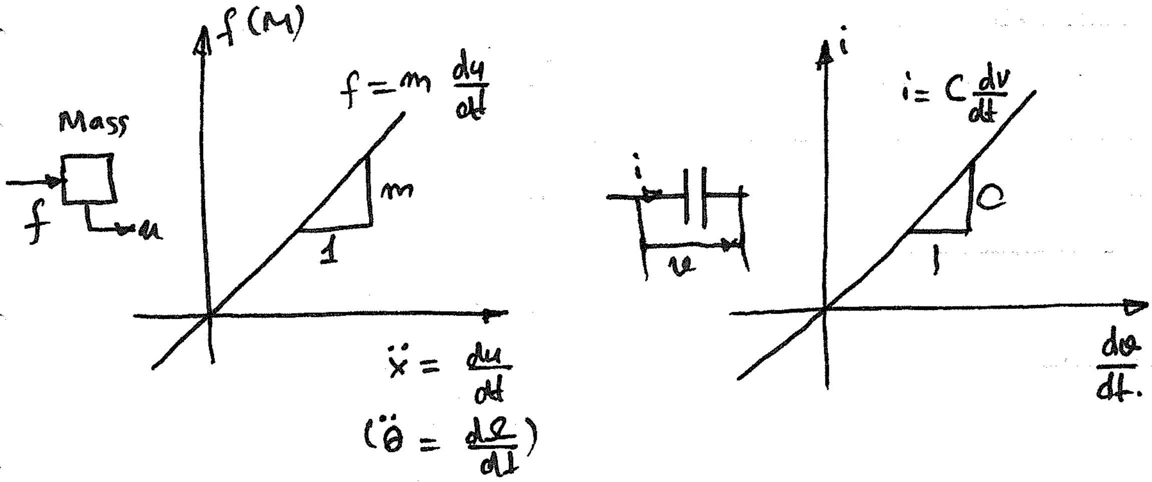
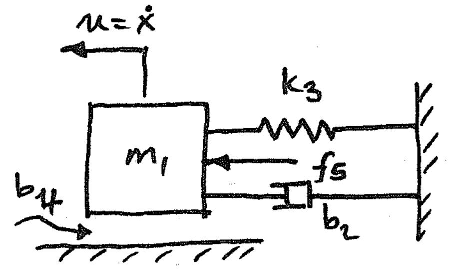


Figure. The physical basis for mechanical-electrical analogy

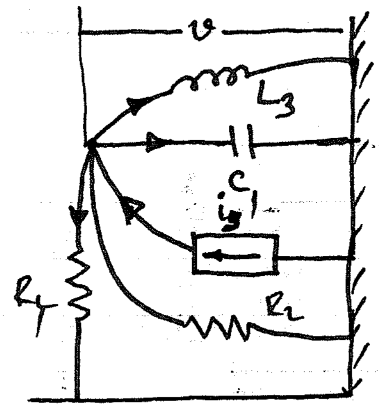
Example: Electrical analog for a simple mechanical system



One degree-of-freedom

$u_1 = u_2 = u_3 = u_4 = u$

f_5 specified

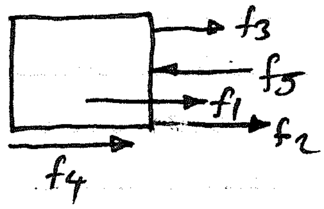


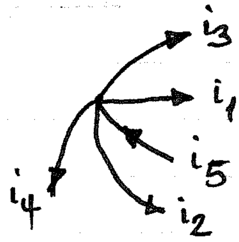
One node; one independent voltage

$v_1 = v_2 = v_3 = v_4 = v$

i_5 specified

Equilibrium





$\sum^{*} f = 0: \; -f_5 + f_1 + f_2 + f_3 + f_4 = 0$

KCL: $-i_5 + i_1 + i_2 + i_3 + i_4 = 0$

Physical relations:

$$f_1 = m_1 \dot{u}_1 \qquad i_1 = C_1 \dot{v}_1$$

$$f_2 = b_2 u_2 \qquad i_2 = \frac{1}{R_2} v_2$$

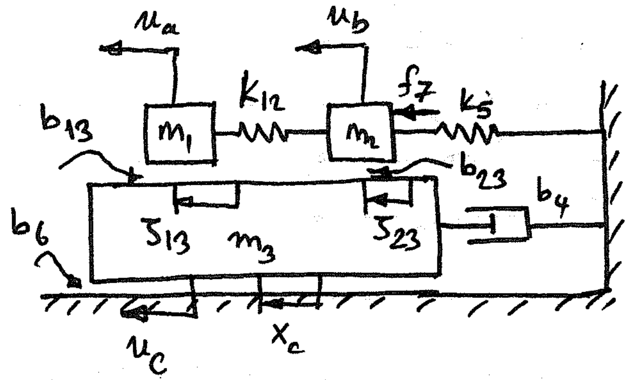
$$f_3 = k_3 \int u_3 dt \qquad i_3 = \frac{1}{L_3} \int v_3 dt$$

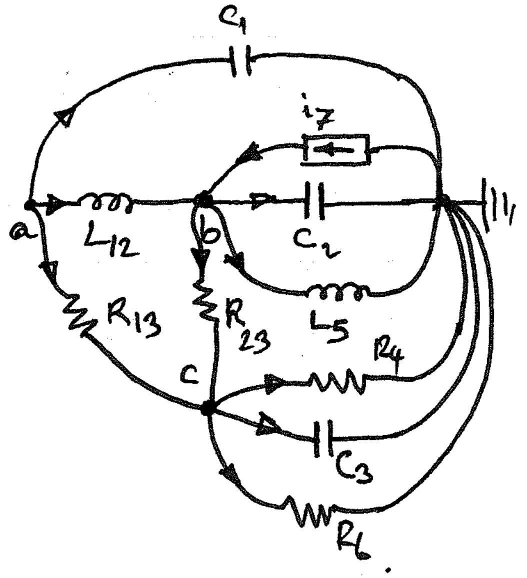
$$f_4 = b_4 u_4 \qquad i_4 = \frac{1}{R_4} v_4$$

$$f_5 = m_1 \dot{u} + b_2 u + k_3 \int u\, dt + b_4 u$$

$$i_5 = C_1 \dot{v} + \frac{1}{R_2} v + \frac{1}{L_3} \int v\, dt + \frac{1}{R_4} v$$

__Example__. A more complex system





Three degree-of-freedom

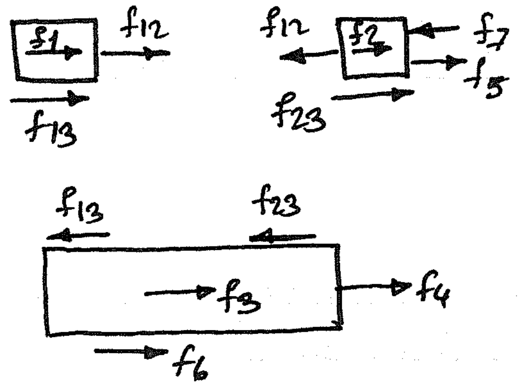
$$\dot{\xi}_{12} = u_a - u_b \qquad u_1 = u_a$$
$$\dot{\xi}_{13} = u_a - u_c \qquad u_2 = u_5 = u_b$$
$$\dot{\xi}_{23} = u_b - u_c \qquad u_3 = u_4 = u_6 = u_c$$

Three independent node voltages

$$v_{12} = v_a - v_b \qquad v_1 = v_a$$
$$v_{13} = v_a - v_c \qquad v_2 = v_5 = v_b$$
$$v_{23} = v_b - v_c\,, \qquad v_3 = v_4 = v_6 = v_c$$

Equilibrium

(i) Free-body diagrams



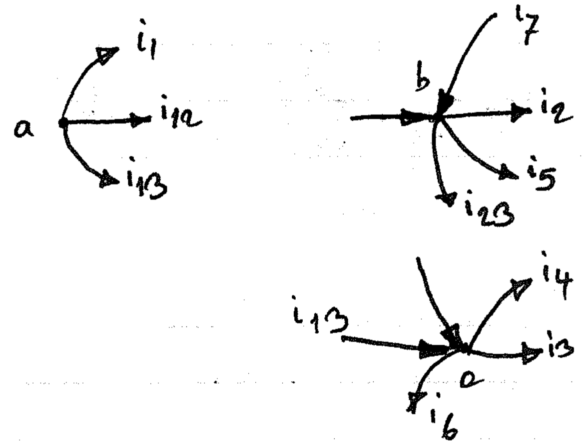
(ii) Force balance ($\sum f^* = 0$)

mass m_1 : $f_1 + f_{12} + f_{13} = 0$

mass m_2 : $-f_7 - f_{12} + f_2 + f_5 + f_{23} = 0$

mass m_3 : $-f_{13} - f_{23} + f_3 + f_4 + f_6 = 0$

(i) Isolate nodes



(ii) KCL ($\sum i_{out} = 0$)

Node a : $i_1 + i_{12} + i_{13} = 0$

Node b : $-i_7 - i_{12} + i_2 + i_5 + i_{23} = 0$

Node c : $-i_{13} - i_{23} + i_3 + i_4 + i_6 = 0$

Physical relations :

$f_2 = m_2 \dot{v}_2$

$f_{23} = b_{23} \dot{\delta}_{23}$

$\vdots$

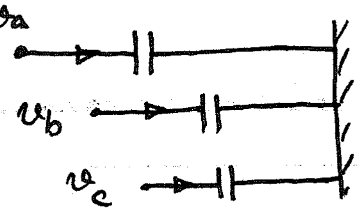
$i_2 = C_2 \dot{v}_2$

$i_{23} = \frac{1}{R_{23}} v_{23}$

$\vdots$

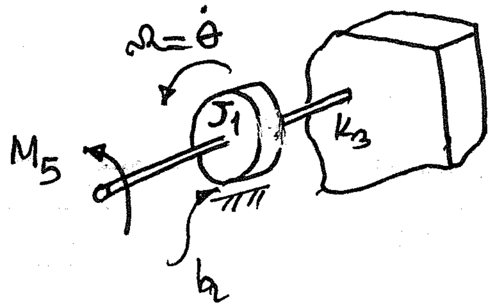
Construction Procedure

- Place first node voltages representing independent velocities and between node voltages and the ground voltages place capacitances representing inertias. Then, distribute other elements between.



"Capacitans, representing masses must always be connected to ground"

Electrical analogy for rotary mechanical systems



$$M_5 = J_1 \dot{\Omega} + b_2 \Omega + k_3 \int \Omega \, dt$$

"Similar to analogy developed for translational mechanical systems"
$J \to m$, $x \to \theta$, $u \to \Omega$ (Substitutions)

Mechanical motions in two or three dimensions :

- Development of electrical analogs more sophisticated !
- Too cumbersome to be useful !

Classification of Dynamic System Elements

Energy storage → T-type : energy storage achieved by virtue of a through variable

Energy storage → A-type : energy storage achieved by virtue of an across variable.

Sources : T or A-types.

Energy - dissipation elements.

Benefits and Limitations of analysis by analogy

- Additional insight into the behavior of a system might be gained by considering its analog
- They prevent us from thinking about new physical phenomena or when they cause us to force a system arbitrarily into a mold of analysis which it does not fit.