EEM 308 Introduction to Communications

Lecture 1

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Course Information

- Lecturer: Asst. Prof. Dr. Can Uysal
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  - TA contact info and office hours will be posted on the webpage.
- Grading:
  - Midterm: 20%
  - Quizzes: 20%
    - There will be 3 – 4 quizzes in total, dates as announced during the course.
  - Labs: 25%
  - Final: 35%
- Textbooks:
  - John G. Proakis, Masoud Salehi - Fundamentals of Communication Systems (2013, Prentice Hall)
  - S. Haykin, Communication Systems, 3rd Ed. New York: Wiley.

Course Information

- Objectives:
  - To introduce the basic principles and techniques used in modern communication systems
  - To allow the student to gain deep analytical and practical understanding of analog modulation principles, transceiver and receiver structures
- Contents:
  - Introduction, Representation of Signals and Systems
  - Frequency Domain Analysis
  - Sampling
  - Hilbert Transforms, Lowpass representations of bandpass signals and systems
  - Amplitude Modulation
    - DSB
    - DSB-SC
    - SSB
  - Angle Modulation
    - FM
    - PM
  - Random Processes
  - Effect of Noise on Analog Communication Systems

Course Information

Laboratory

There will be 6 experiments throughout this semester. There are 6 groups (A - F),

- Exp 1: Intro. to Telecoms Trainer Kit (2 weeks)
- Exp 2: Sampling & Reconstruction
- Exp 3: DSB Modulation & Demodulation
- Exp 4: DSBSC Modulation & Demodulation
- Exp 5: SSB Modulation & Demodulation
- Exp 6: Frequency Modulation & Demodulation
- Lab. dates will be announced later
- Exemption (your last year's grade will be valid)

Figure. EMONA Telecoms-Trainer 101

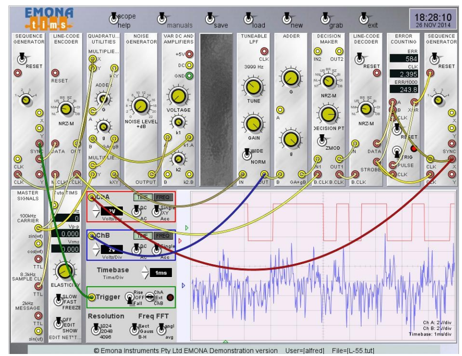


Figure. EMONA Tims Simulator

WHAT'S COMMUNICATIONS

Communication involves the transfer of information from one point to another.

microphone
camera

a speech
picture
music
video
data file

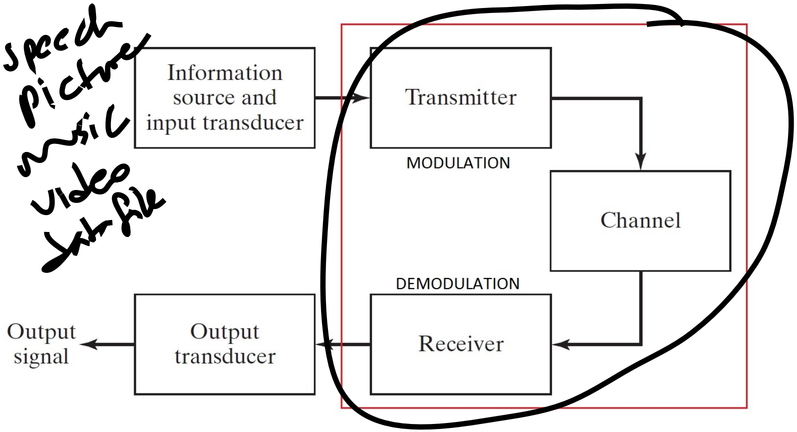


Figure. Functional block diagram of a communication system

Three basic elements:

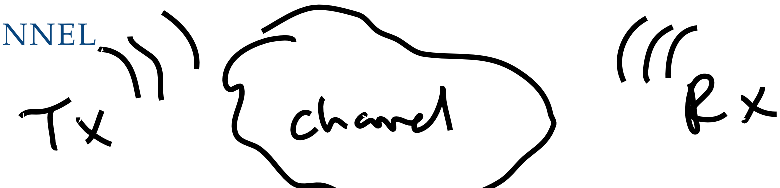
- Transmitter
- Channel
- Receiver

Transmitter

- Converts electrical signal into a form suitable for transmission through the physical channel.
- Need to do this because the transducer output signal cannot, in most cases, be transmitted directly (doesn't match the channel)
- Conversion is made through modulation : amplitude (AM), frequency (FM), phase (PM). Examples AM & FM radio broadcasting
- Other functions: filtering, amplification, radiation

freq
power
bw
antenna size

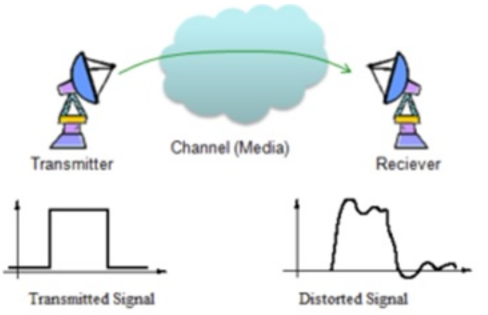
COMMUNICATION CHANNEL



- The physical medium that is used to send the information from the transmitter to the receiver.
- wires: carries the electrical signal Coaxial
  optical fiber: carries the information on a modulated beam
  underwater ocean channel: the information is transmitted acoustically
  free space (air) :information-bearing signal is radiated by use of antennas
- Signal degradations
  - Noise
  - Attenuation
  - Distortion
  - Interference

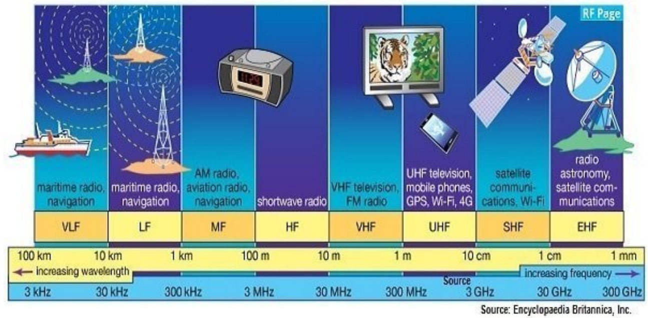
Effects of Channel Response

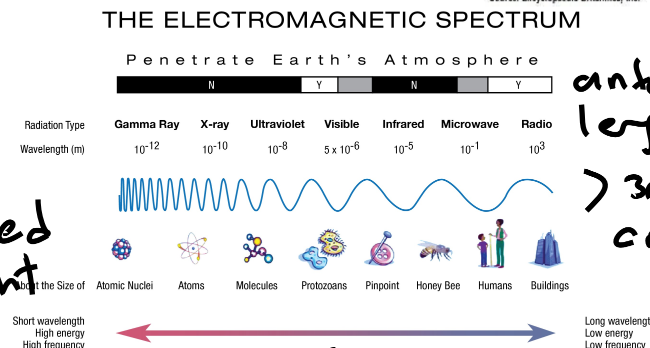
- Attenuation
  - Reduction in signal strength as it propagates through the channel
- Noise
  - Random and unpredictable electrical signals due to natural processes
  - Random motion of electrons (thermal noise)
  - Present in all communication systems
- Distortion
  - Alteration in the shape of waveforms due to imperfect response of the system
- Interference
  - Contamination by signals from other transmitters/signal jammers



WIRELESS (RADIO) COMMUNICATION CHANNELS

- In radio communication systems, electromagnetic energy is coupled to the propagation medium by an antenna which serves as the radiator.
- The physical size and the configuration of the antenna depend primarily on the frequency of operation.
- To obtain efficient radiation of electromagnetic energy, the antenna must be longer than 1/10 of the wavelength.
- Consequently, a radio station transmitting in the FM frequency band, say at 100 MHz (corresponding to a wavelength of $\lambda = \frac{c}{f_c} = 3m$) requires an antenna of at least 30 centimeters.





antenna length > 30 cm

speed of light

FM: 88-108 MHz

$\lambda = \frac{c}{f}$

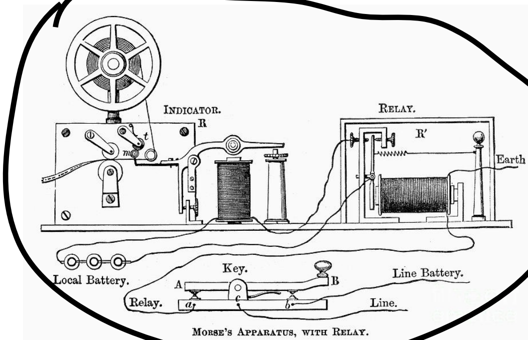
$\lambda = \frac{300 \cdot 10^6 \ m/s}{100 \cdot 10^6 \ 1/s} = 3m$

Receiver

- Main function: to recover the message from the received signal
- Somewhat inverse of the transmitter, but it is greater than the being the inverse
- Also have to eliminate the effects of distortions
- demodulation: inverse of modulation
- Performs other functions such as synchronization to the TX, filtering, suppression of noise & interference

Brief History of Communications

- 1838 - Samuel F. B. Morse demonstrates the telegraph.
- 1876 - Alexander Graham Bell patents the telephone.
- 1897 - Guglielmo Marconi patents a complete wireless telegraph system.
- 1915 - The Bell System completes a U.S. transcontinental telephone line.
- 1925–1927 First television broadcasts in England and the United States.
- 1936 - Regular television broadcasting began by the British Broadcasting Corporation.
- WWII - Radar and microwave systems are developed.

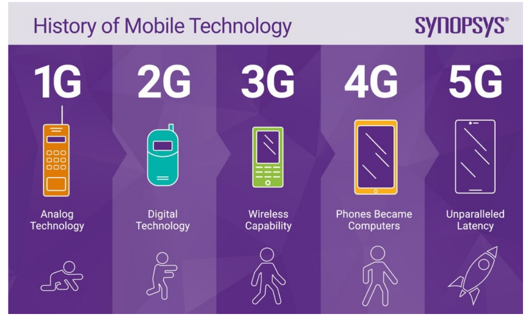


first TV broadcasting in Turkey

TRT - 1968

Brief History of Communications

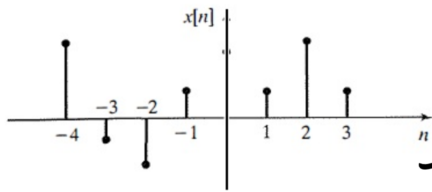
- 1962 - First communications satellite, Telstar I, launched.
- 1966 - First successful facsimile machine.
- 1972 - 1st cellular phone by Motorola
- 1976 - personal computers.
- 1990s - Second generation (2G) cellular systems fielded, Internet, www
- 1995 - Global Positioning System (GPS) reaches full operational capability.
- 1998 - 3G
- 1999 - WiFi
- 2000 - Bluetooth
- 2008 - 4G
- 2019 - 5G

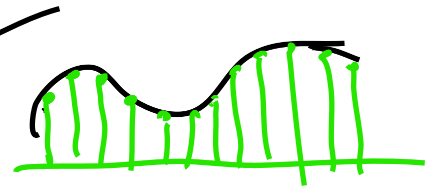


Representation of Signals and Systems

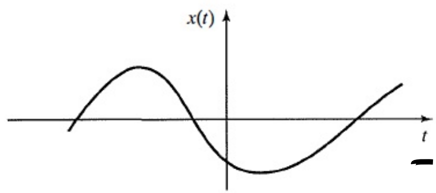
Continuous-Time and Discrete-Time Signals:

- Based on the range of the independent variable, signals can be divided into two classes: continuous-time signals and discrete-time signals
- A continuous-time signal is a signal *x(t)* for which the independent variable t takes real numbers.
- A discrete-time signal, denoted by $x[n]$, is a signal for which the independent variable n takes its values in the set of integers.
- By sampling a continuous-time signal $x(t)$ at time instants separated by T_0, we can define the discrete-time signal $x[n] = x(nT_0)$.





Sampling



$A \cos 2\pi f t$

Deterministic and Random Signals:

- In a deterministic signal at any time instant t, the value of $x(t)$ is given as a real or a complex number.
- In a random (or stochastic) signal at any given time instant t, $x(t)$ is a random variable; i.e., it is defined by a probability density function.

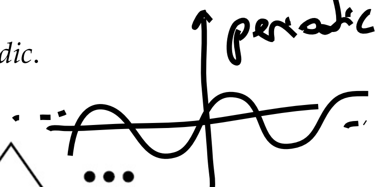
Periodic and Nonperiodic Signals:

- A signal g is called periodic if it repeats in time; i.e., for some $T_0 > 0$

$$g(t + T_0) = g(t) \qquad \textit{for all } t \tag{1}$$

  T_0 is the period of the signal
- A signal that does not satisfy the periodicity conditions is called *nonperiodic*.

periodic



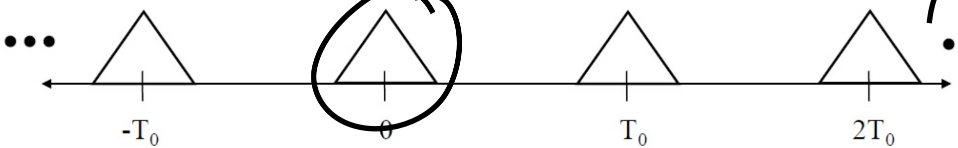
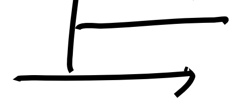


Figure. A periodic signal

T_0

$u(t)$ nonperiodic



Even and Odd Signals:

- Evenness and oddness are expressions of various types of symmetry present in signals.
- A signal $x(t)$ is even if it has mirror symmetry with respect to the vertical axis.
- A signal is odd if it is anti-symmetric with respect to the vertical axis.

The signal $x(t)$ is even if and only if, for all t,

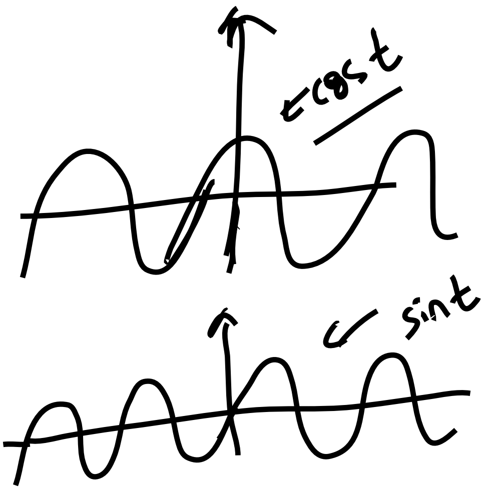
$$x(-t) = x(t), \tag{2}$$

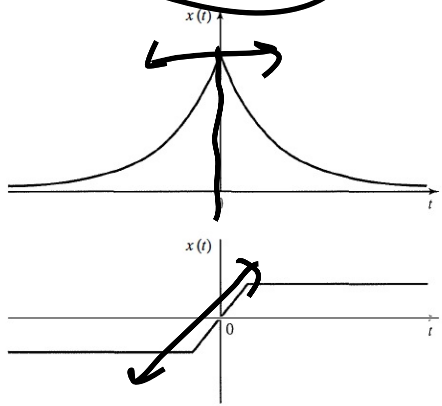
$\cos(-t) = \cos t$

and is odd if and only if, for all t,

$$x(-t) = -x(t). \tag{3}$$

$\sin(-t) = -\sin t$





← even

← odd

Representation of Signals and Systems

This classification deals with the energy content and the power content of signals. Before classifying these signals, we need to define the energy content (or simply the energy) and the power content (or power).

Energy-Type and Power-Type Signals:

energy
digital comm

The energy content of a signal $x(t)$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

power
analog comm

The power content of a signal $x(t)$

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

- A signal is energy-type if $E_x < \infty$
- A signal is power-type if $0 < P_x < \infty$ (finite and nonzero)
- A signal cannot be both power-type and energy-type. Why?
- A signal can be neither energy-type nor power-type

Some Important Signals and Their Properties

sine wave ✗

Sinusoidal Signal: The sinusoidal signal is defined by

cos wave

$$x(t) = A cos(2\pi f_0 t + \theta), \tag{4}$$

where the parameters A, f_0, and θ are, respectively, the amplitude, frequency, and phase of the signal. A sinusoidal signal is periodic with the period $T_0 = 1/f_0$.

The Complex Exponential Signal: The complex exponential signal is defined by

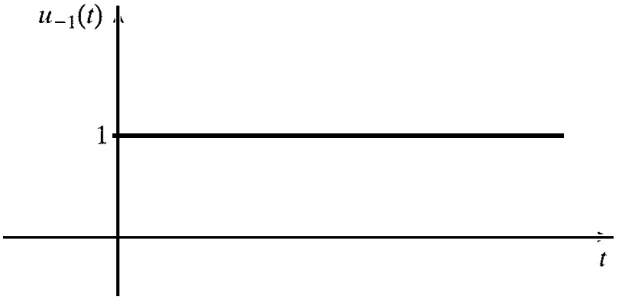
$$x(t) = Ae^{j(2\pi f_0 t+\theta)}. \tag{5}$$

The Unit-Step Signal (function):

u(t)

$$u_{-1}(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

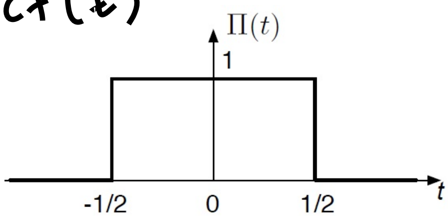
The unit step function corresponds to turning on at time 0

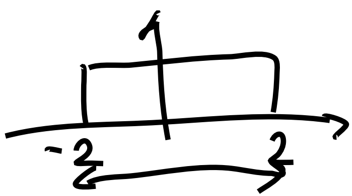


Rectangular Pulse (Unit Rectangle): This signal is defined as

$$\Pi(t) = \begin{cases} 1 & -1/2 \leq t \leq 1/2 \\ 0 & \textit{otherwise} \end{cases} \tag{6}$$

rect(t)

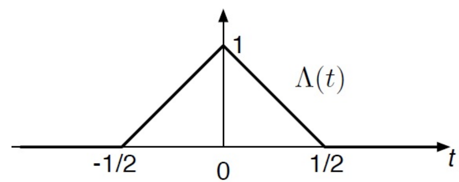


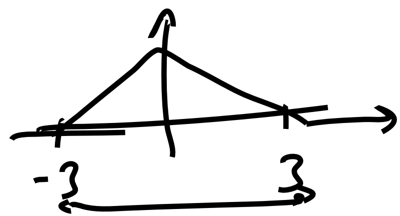


$\Pi\left(\frac{t}{4}\right)$ t

Triangular Signal (Unit Triangle): This signal is defined as

$$\Lambda(t) = \begin{cases} |t| & |t| \leq 1/2 \\ 0 & \textit{otherwise} \end{cases} \tag{7}$$





$\Lambda\left(\frac{t}{6}\right)$ t

Example:

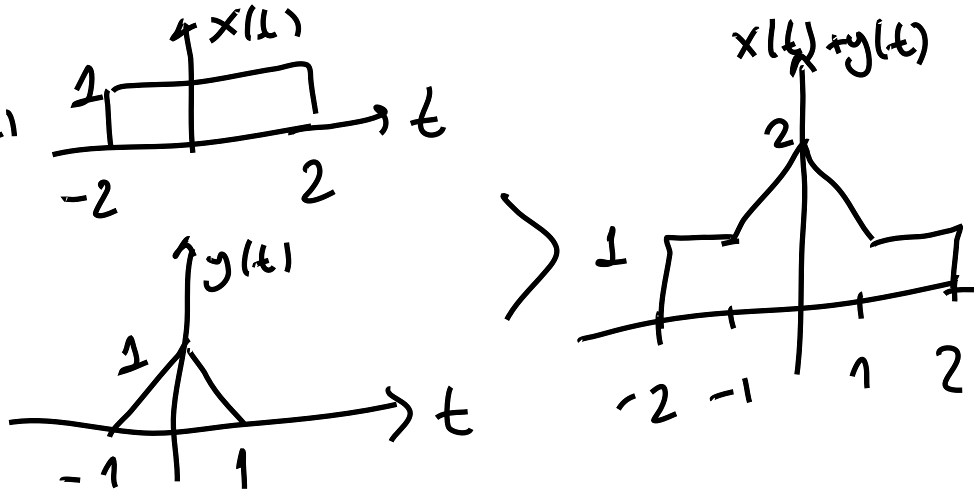
rect

y(t)

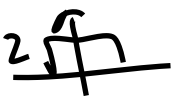
Plot $\Pi(\frac{t}{4}) + \Lambda(\frac{t}{2})$

tri

x(t)



$2\Pi(\frac{t}{4})$



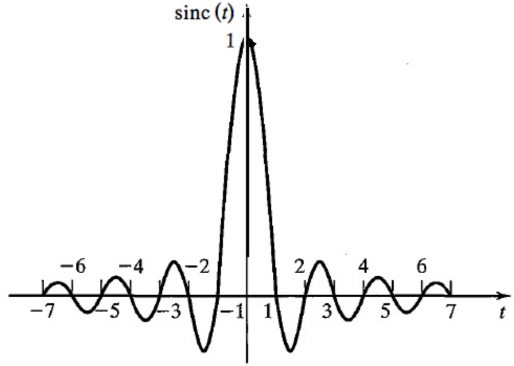
Some Important Signals and Their Properties

Sinc Signal:

$\frac{\sin(\pi t)}{\pi t}$

The sinc signal is defined as

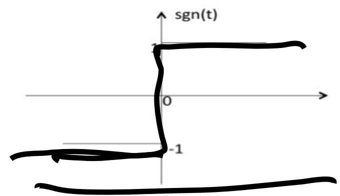
$$\operatorname{sinc}(t) = \begin{cases} \frac{\sin(\pi t)}{\pi t} & t \neq 0 \\ 1 & t = 0 \end{cases} \tag{8}$$



From this figure, we can see that the sinc signal achieves its maximum of 1 at $t = 0$. The zeros of the sinc signal are at $t = \pm 1, \pm 2, \pm 3, \ldots$.

Sign or Signum Signal: The sign or the signum signal denotes the sign of the independent variable t and is defined by

$$sgn(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases} \tag{9}$$

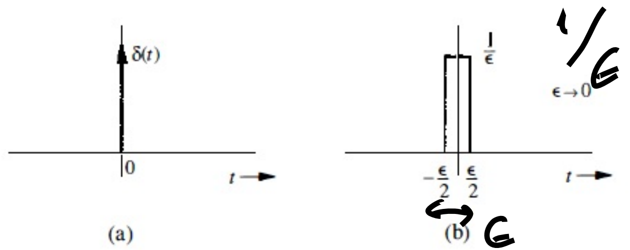


SOME IMPORTANT SIGNALS AND THEIR PROPERTIES

Unit Impulse or Delta Signal: The unit impulse function $\delta(t)$ is one of the most important functions in the study of signals and systems.
The unit impulse function $\delta(t)$ was first defined by P. A. M. Dirac (hence often known as the "Dirac delta") as

$$\begin{aligned} \delta(t) &= 0, \quad t \neq 0 \\ \int_{-\infty}^{\infty} \delta(t) dt &= 1 \end{aligned} \tag{10}$$

We can visualize an impulse as a tall, narrow rectangular pulse of unit area. The width of this rectangular pulse is a very small value ϵ; its height is a very large value $1/\epsilon$ in the limit as ϵ –> 0.



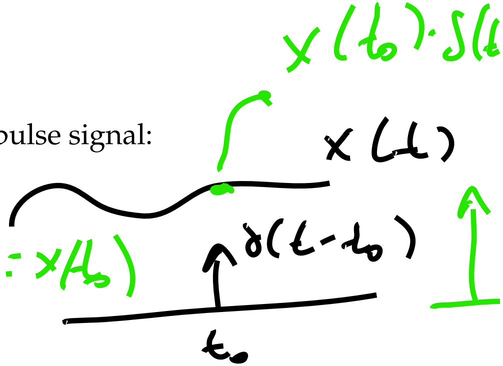
Some properties of Delta Function

The following properties are derived from the definition of the impulse signal:

1. $\delta(t) = 0$ for all $t \neq 0$ and $\delta(0) = \infty$
2. $x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$
3. For any $\phi(t)$ continuous at t_0

$$\int_{-\infty}^{\infty} \phi(t)\delta(t - t_0)dt = \phi(t_0)$$

$\int x(t)\,\delta(t-t_0) = x(t_0)$



4. The result of the convolution of any signal with the impulse signal is the signal itself:

$$x(t) \star \delta(t) = x(t)$$

Also,

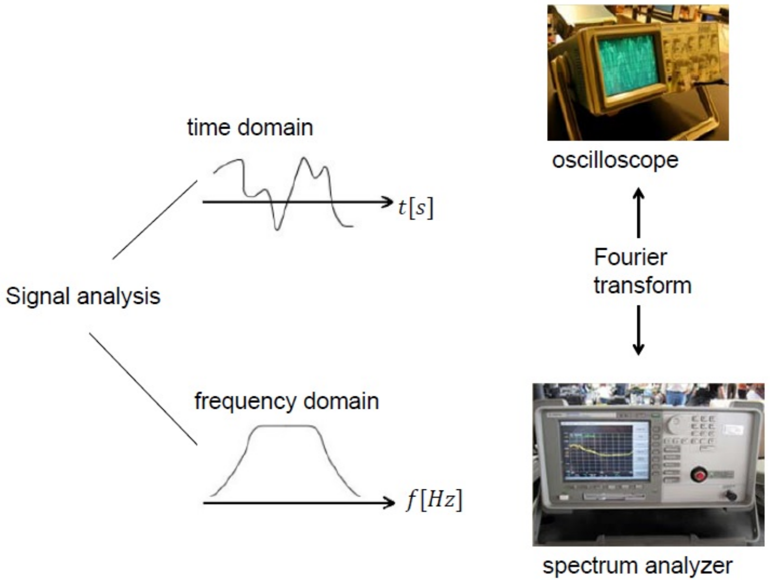
$$x(t) \star \delta(t - t_0) = x(t - t_0)$$

5. For all $a \neq 0$,

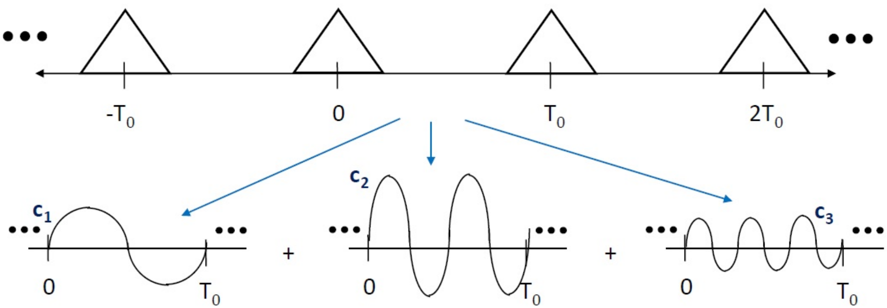
$$\delta(at) = \frac{1}{|a|}\delta(t)$$

Frequency Domain Analysis

- Analysis of signals and system is easier (most of the time) in frequency domain
- Tools for frequency domain analysis are: Fourier Series (for periodic signals) and Fourier Transform (for nonperiodic signals)



Fourier Series



Periodic signals can be expressed as a sum of sinusoidal components with frequencies f_0, $2f_0$, $3f_0$, …, kf_0 and complex weights c_0, c_1, c_2, …, c_k

- Jean-Baptiste Joseph Fourier: 1768-1830
- Student of Laplace and Lagrange
- 1807: introduced the Fourier series expansion

FOURIER SERIES

T_0: period

- Periodic signals can be written as the sum of sinusoids whose frequencies are integer multiples of the *fundamental frequency* $\boxed{f_0 = 1/T_0.}$
- The most general representation uses complex exponential functions.

$$g(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi f_0 n t}$$

In general, the Fourier series coeffcients C_n are complex numbers, even when the signal is real valued. Note the factor of 2π since we are using the frequency in cycles/second, or Hz.

- The Fourier series coefficents can be computed by

$$C_n = \frac{1}{T_0} \int_a^{a+T_0} g(t) e^{-j2\pi f_0 n t}\, dt$$

a: arbitrary

The integral is over any period of the signal.

Fourier Series

- $f_0 = \frac{1}{T_0}$, fundamental frequency
- frequencies of complex exponential signals are multiples of f_0. The *nth* multiple of f_0 is called the *nth* harmonic.
- the periodic signal g(t) can be described by the period T_0 (or the fundamental frequency f_0) and the sequence of complex numbers C_n.
- The Fourier-series expansion can be expressed in terms of the angular frequency $w_0 = 2\pi f_0$ by

$$C_n = \frac{w_0}{2\pi} \int_{\alpha}^{\alpha + \frac{2\pi}{w_0}} g(t) e^{-jnw_0 t} dt \tag{11}$$

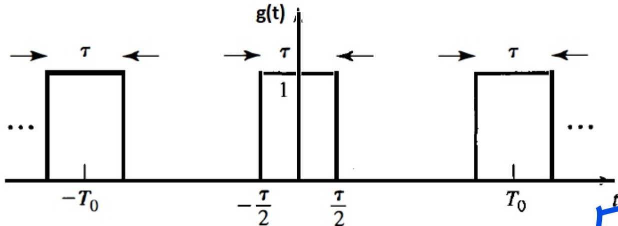
$$g(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jnw_0 t} \tag{12}$$

EXAMPLE

Let $g(t)$ denote the periodic signal depicted in the figure and described analytically by

$$g(t) = \sum_{n=-\infty}^{+\infty} \Pi\left(\frac{t - nT_0}{\tau}\right)$$

where τ is a given positive constant (pulse width). Determine the Fourier-series expansion for this signal.



T_0: period

τ: pulse width

$$C_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g(t) e^{-jn\frac{2\pi}{T_0}t}$$

$$= \frac{1}{T_0} \int_{-\tau/2}^{\tau/2} 1 e^{-jn\frac{2\pi}{T_0}t} dt$$

$$\boxed{\int e^{cx} dx = \frac{1}{c} e^{cx}}$$

Example: Solution

$$= \frac{1}{-jn\frac{2\pi}{\cancel{T_0}}} \cdot \frac{1}{\cancel{T_0}} e^{-jn\frac{2\pi}{T_0}t} \Bigg|_{t=-\tau/2}^{t=\tau/2}$$

$$= \frac{1}{-jn2\pi} \underbrace{\left[e^{-jn\frac{\cancel{2}\pi}{T_0}\frac{\tau}{\cancel{2}}} - e^{-jn\frac{\cancel{2}\pi}{T_0}\left(-\frac{\tau}{\cancel{2}}\right)} \right]}_{\sin\left(\frac{n\pi\tau}{T_0}\right)\cdot -2j}$$

$$\boxed{\sin x = \frac{e^{jx} - e^{-jx}}{2j}}$$

$$= \frac{\sin\left(\frac{n\pi\tau}{T_0}\right)\cdot \cancel{-2j}}{\cancel{-j}\,n\,\cancel{2}\pi} = \frac{\sin\left(\frac{n\pi\tau}{T_0}\right)}{n\pi}$$

$$\boxed{\text{sinc}\, x = \frac{\sin(\pi x)}{\pi x}}$$

$$= \frac{2}{T_0} \frac{\sin\left(\frac{n\pi 2}{T_0}\right)}{n\pi \frac{2}{T_0}} = \frac{2}{T_0}\,\text{sinc}\left(\frac{\pi 2}{T_0}\right)$$

$$g(t) = \sum_{n=-\infty}^{\infty} \frac{2}{T_0}\,\text{sinc}\left(\frac{\pi 2}{T_0}\right) e^{jn\frac{2\pi}{T_0}t}$$

Fourier Series for Real Signals

- There are alternative ways to expand the real signal $x(t)$.
- For $x(t)$, we have $x_{-n} = x_n^*$

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(2\pi \frac{n}{T_0} t\right) + b_n \sin\left(2\pi \frac{n}{T_0} t\right) \right]$$

← Trigonometric FS

$$a_n = \frac{2}{T_0} \int_{\alpha}^{\alpha+T_0} x(t) \cos\left(2\pi \frac{n}{T_0} t\right) dt$$

$$b_n = \frac{2}{T_0} \int_{\alpha}^{\alpha+T_0} x(t) \sin\left(2\pi \frac{n}{T_0} t\right) dt$$

$$\boxed{x_n = \frac{a_n - jb_n}{2}}$$

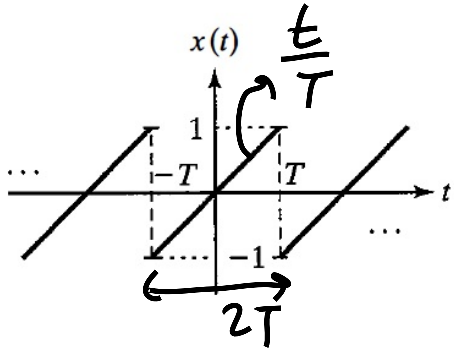
- This relation, which only holds for real periodic signals, is called the trigonometric FS expansion.
- A third way exists to represent the Fourier-series expansion of a real signal

$$x(t) = x_0 + 2 \sum_{n=1}^{\infty} |x_n| \cos\left(2\pi \frac{n}{T_0} t + \angle x_n\right)$$

$$|x_n| = \frac{1}{2}\sqrt{a_n^2 + b_n^2}, \quad \angle x_n = -\arctan\left(\frac{b_n}{a_n}\right)$$

Example: Fourier Series

Determine the Fourier-series expansion of the given signal (assume $T_0 = 2$)



$$T_0 = 2 = 2T$$
$$T = 1$$

$$a_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} \frac{t}{T} \cos\left(2\pi \frac{n}{T_0} t\right) dt$$

$$b_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} \frac{t}{T} \sin\left(2\pi \frac{n}{T_0} t\right) dt$$

EXAMPLE: SOLUTION

$$a_n = \int_{-1}^{1} t\cos(\pi n t)\,dt$$

$$\boxed{\int x\cos(px)\,dx = \frac{1}{p^2}\cos(px) + \frac{x}{p}\sin px}$$

$$= \frac{1}{\pi^2 n^2}\cos\pi n t + \frac{t}{\pi n}\sin\pi n t \Big|_{t=-1}^{t=1}$$

$$= \left(\frac{1}{\pi^2 n^2}\cos\pi n + \frac{1}{\pi n}\sin\pi n\right) - \left(\frac{1}{\pi^2 n^2}\underbrace{\cos(-\pi n)}_{\cos\pi n} - \frac{1}{\pi n}\underbrace{\sin(-\pi n)}_{-\sin(\pi n)}\right)$$

$$a_n = 0$$

$$b_n = \int_{-1}^{1} t\sin(\pi n t)\,dt =$$

$$\boxed{\int x\sin(px) = \frac{1}{p^2}\sin(px) - \frac{x}{p}\cos px}$$

$$b_n = \frac{1}{\pi^2 n^2}\sin \pi n t - \frac{t}{\pi n}\cos \pi n t \Big|_{t=-1}^{t=1}$$

EXAMPLE: SOLUTION-CONT'D

$$b_n = \left[\left(\frac{1}{\pi^2 n^2}\sin \pi n - \frac{1}{\pi n}\cos \pi n\right) - \left(\frac{1}{\pi^2 n^2}\sin(-\pi n) + \frac{1}{\pi n}\cos(-\pi n)\right)\right]$$

$$= \frac{2}{\pi^2 n^2}\sin \pi n - \frac{2}{\pi n}\cos \pi n = \frac{2(\sin \pi n - \pi n \cos \pi n)}{\pi^2 n^2}$$

using $\sin(\pi n) = 0$
$\cos(\pi n) = (-1)^n$

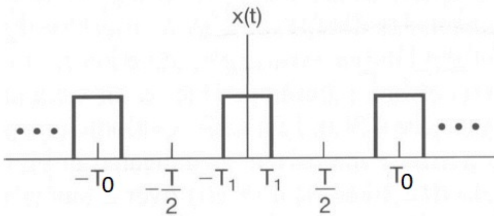
$$= \frac{-2\pi n(-1)^n}{\pi^2 n^2} = \frac{-2}{\pi n}(-1)^n$$

$$x(t) = \sum_{n=1}^{\infty} b_n \sin\left(2\pi \frac{n}{T_0} t\right) = \sum_{n=1}^{\infty} -\frac{2}{\pi n}(-1)^n \sin \pi n t$$

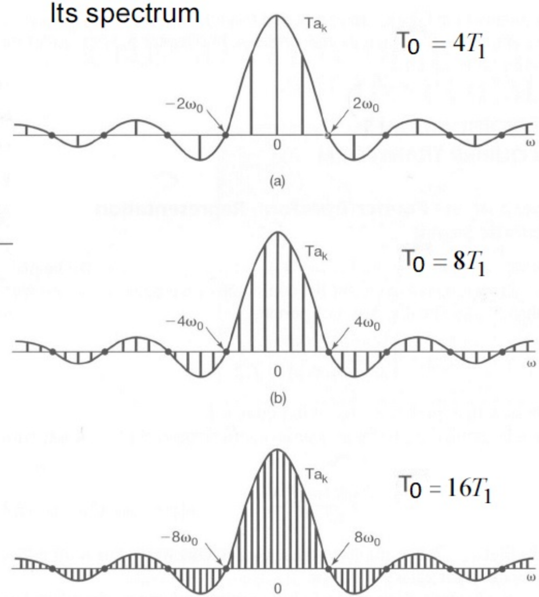
FOURIER TRANSFORM

- Frequency domain representation of nonperiodic signals can be obtained using Fourier Transform
- Fourier Transform and Inverse Fourier Transform provides a one-to-one mapping between time domain and frequency domain
- Nonperiodic signal can be considered as periodic with T_0 -> ∞

Periodic signal



Its spectrum



As T_0 increases, spectral components are getting closer and closer, becoming the continuous spectrum at the limit

Fourier Transform

- The Fourier transform (or Fourier integral) of $x(t)$, defined by

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft}dt \tag{13}$$

exists and the original signal can be obtained from its Fourier transform by

$$x(t) = \int_{-\infty}^{+\infty} X(f)e^{j2\pi ft}dt \tag{14}$$

- Note that these are almost completely symmetric. Only the sign of the complex exponential changes.
- This will simplify a lot of the transforms and theorems.
- To denote that $X(f)$ is the Fourier transform of $x(t)$, we frequently employ the following notation:

$$X(f) = \mathcal{F}\{x(t)\}. \tag{15}$$

- To denote that $x(t)$ is the inverse Fourier transform of $X(f)$, we use the following notation:

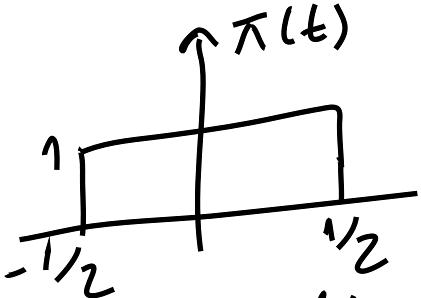
$$x(t) = \mathcal{F}^{-1}\{X(f)\} \tag{16}$$

- in short

$$x(t) \iff X(f) \tag{17}$$

Example: Fourier Transform

Determine the Fourier Transform of the signal given below,



$$\Pi(t) = \begin{cases} 1 & -1/2 \leq t \leq 1/2 \\ 0 & \textit{otherwise} \end{cases} \tag{18}$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

$$\mathcal{F}\{\Pi(t)\} = \int_{-1/2}^{1/2} 1 \, e^{-j2\pi ft} dt$$

$$= \frac{1}{-j2\pi f} e^{-j2\pi ft} \Big|_{t=-1/2}^{t=1/2}$$

Example: Solution

$$= \frac{1}{-j2\pi f}\Big[\underbrace{e^{-j\pi f} - e^{j\pi f}}_{-2j\sin \pi f}\Big] = \frac{-\cancel{2j}\sin \pi f}{-\cancel{j2}\pi f}$$

$$= \frac{\sin \pi f}{\pi f} = \mathrm{sinc}(f)$$

$$\Pi(t) \Longleftrightarrow \mathrm{sinc}(f)$$

Basic Properties of FT

- Linearity: The Fourier Transform operation is linear

$$x_1(t) \Leftrightarrow X_1(f) \quad \text{and} \quad x_2(t) \Leftrightarrow X_2(f)$$
$$\alpha x_1(t) + \beta x_2(t) \Leftrightarrow \alpha X_1(f) + \beta X_2(f)$$

  where α and β are two arbitrary scalars
- Duality: If $X(f) = \mathcal{F}[x(t)]$,

$$x(f) = \mathcal{F}[X(-t)] \quad \text{and} \quad x(-f) = \mathcal{F}[X(t)]$$

- Time shift:

$$\mathcal{F}\left[x(t - t_0)\right] = e^{-j2\pi f t_0}\mathcal{F}[x(t)]$$

- Scaling: For any real $a \neq 0$

$$\mathcal{F}[x(at)] = \frac{1}{|a|}X\left(\frac{f}{a}\right)$$

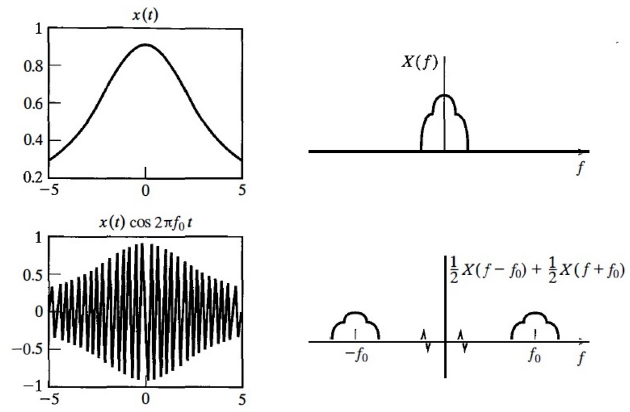
- Convolution: If $x(t) \Leftrightarrow X(f)$ and $y(t) \Leftrightarrow Y(f)$

$$\mathcal{F}[x(t) * y(t)] = \mathcal{F}[x(t)] \cdot \mathcal{F}[y(t)] = X(f)Y(f)$$

Basic Properties of FT

- Modulation:

$$x(t)\cos(2\pi f_c t) \Leftrightarrow \frac{1}{2}\left[X(f-f_c)+X(f+f_c)\right]$$



Basic Properties of FT

- Parseval's property:

$$\int_{-\infty}^{\infty} x(t)y^*(t)dt = \int_{-\infty}^{\infty} X(f)Y^*(f)df$$

- Rayleigh's property:

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

FT OF PERIODIC SIGNALS

Let $x(t)$ be a periodic signal with period T_0

$$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{j2\pi \frac{n}{T_0} t} \tag{19}$$

- By taking the Fourier transform of both sides we obtain

$$X(f) = \sum_{n=-\infty}^{\infty} x_n \delta(f - \frac{n}{T_0}) \tag{20}$$

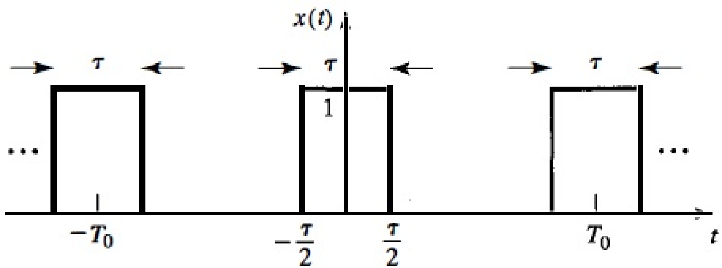
- From this relation, we observe that the Fourier transform of a periodic signal $x(t)$ consists of a sequence of impulses in frequency at multiples of the fundamental frequency of the periodic signal. The weights of the impulses are simply the Fourier-series coefficients of the periodic signal.
- Let $x_{T_0}(t)$ be one period of $x(t)$ and $x_{T_0}(t) \iff X_{T_0}(f)$

$$x_n = \frac{1}{T_0} X_{T_0}(f)\Big|_{f=\frac{n}{T_0}} = \frac{1}{T_0} X_{T_0}\left(\frac{n}{T_0}\right) \tag{21}$$

A shortcut for computing FS coefficients!

Example

Determine the Fourier series coefficients of the signal $x(t)$, as shown in Figure



$\Pi\left(\frac{t}{\tau}\right)$

$$\mathcal{F}\left\{\Pi\left(\frac{t}{\tau}\right)\right\} = \int_{-\tau/2}^{\tau/2} 1\, e^{-j2\pi f t}\, dt = \frac{1}{-j2\pi f} e^{-j2\pi f t}\Big|_{t=-\tau/2}^{t=\tau/2}$$

$$= \frac{1}{-j2\pi f}\left[e^{-j\pi f\tau} - e^{j\pi f\tau}\right] = \frac{\tau}{\tau}\,\frac{\sin \pi f\tau}{\pi f} = \tau\, \mathrm{sinc}(f\tau)$$

Power and Energy

$x_n = \frac{1}{T_0} X_{T_0}\left(\frac{n}{T_0}\right)$ $X(f) = 2\,\mathrm{sinc}(f2)$

$X\left(\frac{n}{T_0}\right) = 2\,\mathrm{sinc}\left(\frac{n2}{T_0}\right)$

$x_n = \frac{2}{T_0}\,\mathrm{sinc}\left(\frac{n2}{T_0}\right)$ //

The energy and power of a signal represent the energy or power delivered by the signal when it is interpreted as a voltage or current source feeding a 1 Ω resistor.
Energy Content:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 d(t) \tag{22}$$

Power content:

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 d(t) \tag{23}$$

- A signal is energy-type if $E_x < \infty$
- A signal is power-type if $0 < P_x < \infty$
- A signal cannot be both power-type and energy-type.
- A signal can be neither energy-type nor power-type